NONISOTHERMAL FLOW OF A NON-NEWTONIAN FLUID IN THE SLOWLY CONVERGING CHANNEL OF A BELT WORM-CONVEYOR PUMP UNDER CONDITIONS OF COMPLEX SHEAR

V. I. Yankov, V. I. Boyarchenko, and A. L. Krylov

UDC 532.135:536.22

The flow of an anomalously viscous fluid with an exponential rheological equation in the converging channel of the screw of a belt worm-conveyor pump is examined taking into account the circulatory flow of the fluid, the dissipation of mechanical energy, and the exchange of heat with the environment.

The subject of [1] is the isothermal flow of a non-Newtonian fluid with an exponential rheological equation in the convergent channel of a worm-conveyor pump taking into account the complex shear nature of the flow. The problem of determining the pressure gradient in any channel cross section is shown to be reducible to a set of four transcendental equations. Insofar as the isothermal mode of the flow is purely hypothetical, since mechanical energy dissipation always occurs in the screw channel, it is interesting to examine a similar problem in a nonisothermal formulation.

Let us examine the laminar transient nonisothermal flow of a non-Newtonian fluid in the slowly converging channel of a belt worm-conveyor pump, represented by a coil of trapezoidal cross section, rotating with small clearances relative to the fixed outer cylinder and the inner truncated cone (Fig. 1a).

In the examination of this problem it is assumed that there are no clearances between the ridges of the screw and the housing and that the initial depth of the channel H is less than its width S and much less than the radius of the screw. The plane model of the screw channel is, therefore, used (Fig. 1b) and, for greater clarity, the motion is reversed. The x axis is aligned along the channel, the y axis along its width, and the z axis along the depth of the channel. The lower plate is inclined at an angle δ to the x axis and moves at a velocity V_l, while the upper plate is parallel with the x axis and moves at a velocity V_u. Pressure gradients $\partial p/\partial x = A_1$ and $\partial p/\partial y = A_2$ move along the x and y axes. The slope of the plates in the direction of the y axis and the component of fluid velocity in the direction of the z axis will be neglected.

The velocity V_l will be taken to imply the velocity at the mean radius of the truncated cone. In addition, it will be assumed that the heat transfer between the product and the ambient proceeds in accordance with Newton's law with a coefficient of heat transfer α, and the product temperature due to intensive mixing varies only along the channel, remaining constant in the two other directions. It is also assumed that the specific heat of the product C varies linearly with a rise in temperature $C = C_0 + \Omega T$ and that its viscosity varies in accordance with the Reynolds law $B = B_1/\exp(kT)$.

With a certain product flow rate Q_1 there may be a cross section $h_x = h_0$ in which $A_1 = 0$ in the channel. In the $H \geq h_x \geq h_0$ channel cross section, therefore, $A_1 \geq 0$ and in the $h \leq h_x \leq h_0$ cross section, $A_1 \leq 0$ (h is the finite channel depth). The pressure gradient A_2 is always positive, since the flow rate in the direction of the y axis is equal to zero.

All-Union Synthetic Fiber Scientific-Research Institute, Kalinin; Institute of Chemical Physics Division, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 30, No. 6, pp. 1040-1046, June, 1976. Original article submitted March 25, 1975.

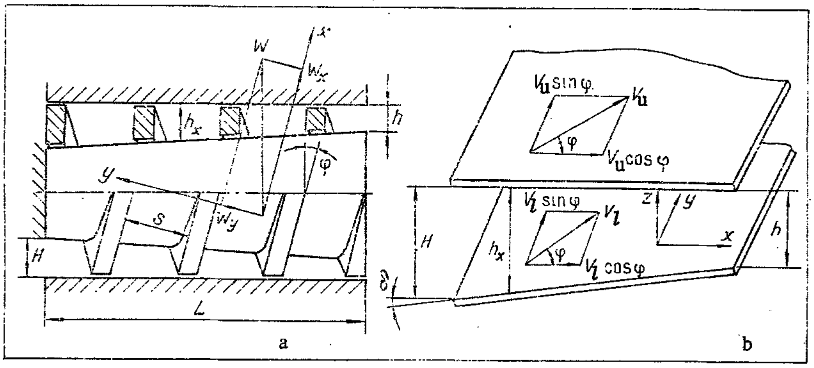

Fig. 1. Diagram of belt worm-conveyor pump with converging channel (a) and plane model of it (b).

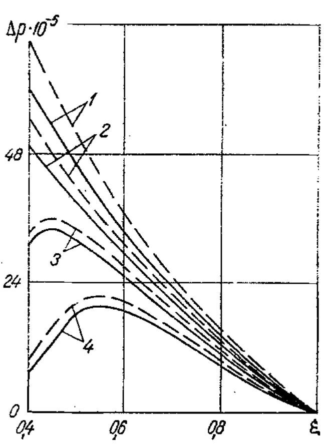

Fig. 2

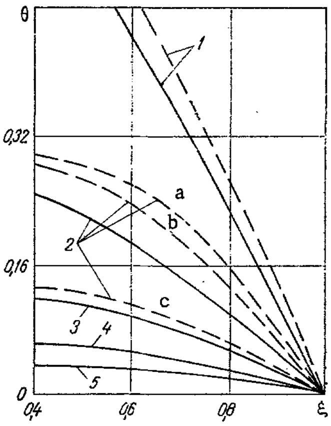

Fig. 3

Fig. 2. Distribution of pressure p (N/m^2) along the length of channel for different flow rate values Q_1 (m^3/sec): 1) $Q_1 = 1 \cdot 10^{-4}$; 2) $1.5 \cdot 10^{-4}$; 3) $2 \cdot 10^{-4}$; 4) $2.5 \cdot 10^{-4}$.

Fig. 3. Distribution of dimensionless temperature along length of screw channel for different flow rate values Q_1 (m^3/sec): 1) $Q_1 = 0.5 \cdot 10^{-4}$; 2) $1 \cdot 10^{-4}$; 3) $1.5 \cdot 10^{-4}$; 4) $2 \cdot 10^{-4}$; 5) $2.5 \cdot 10^{-4}$; b) $T_0 = 130°C$; $\alpha = 500$ J/m^2 · sec · deg; a) $T_0 = 130°C$; $\alpha = 1000$; c) $T_0 = -10°C$, $\alpha = 500$.

The inclined plane is replaced by a series of small steps dx long and parallel to the axis, and it is assumed that the product flow within the limits of each such step proceeds in the same way as between the parallel planes.

The generalized exponential law [2, 3], in which the constants B_1, k, and n are independent of the product temperature, is used as the rheological equation:

$$\bar{\tau} = \frac{B_1}{\exp(kT)} \left[\left(\frac{\partial W_x}{\partial z} \right)^2 + \left(\frac{\partial W_y}{\partial z} \right)^2 \right]^{\frac{n-1}{2}} \bar{\Delta}. \tag{1}$$

The equations of motion in projections on the x and y axes take the form

$$\partial \tau_{xz} / \partial z = A_1, \quad \partial \tau_{yz} / \partial z = A_2, \tag{2}$$

and their solution will be

$$\tau_{xz} = A_1 z - A_0 h_x c_1, \; \tau_{yz} = A_2 z - A_0 h_x c_2. \tag{3}$$

The following dimensionless magnitudes are introduced in solving this problem:

$$\zeta = z/h_x, \; v_1 = W_x/V_u, \; v_2 = W_y/V_u, \; a_1 = A_1/A_0,$$

$$a_2 = A_2/A_0, \quad e = V_l/V_u, \quad \xi = h_x/H, \quad \xi_0 = h_0/H, \quad b = h/H, \tag{4}$$

$$\theta = \frac{(T - T_i)}{n} k, \quad \theta_c = \frac{(T_f - T_i)}{n} k, \quad q_1 = \frac{Q_1}{V_u h_x Si},$$

$$\beta = \left[\frac{A_0 \exp(kT_i)}{B_1}\right]^{\frac{1}{n}} \frac{h_x^{\frac{1+n}{n}}}{V_u}, \quad q_2 = \frac{Q_2}{V_u h_x li}.$$

The combined solution of Eqs. (1) and (2), taking into account the dimensionless magnitudes of (4), the adhesion conditions, and the conditions imposed on the product flow rates q_1 and q_2, is written in the form [4]

$$v_1 = \beta \exp\theta \int_0^{\zeta} G(a_1\zeta - c_1)\, d\zeta + e\cos\varphi, \tag{5}$$

$$v_2 = \beta \exp\theta \int_0^{\zeta} G(a_2\zeta - c_2)\, d\zeta + e\sin\varphi, \tag{6}$$

$$\beta \exp\theta \int_0^1 G(a_1\zeta - c_1)\, d\zeta - (1 - e)\cos\varphi = 0, \tag{7}$$

$$\beta \exp\theta \int_0^1 G(a_2\zeta - c_2)\, d\zeta - (1 - e)\sin\varphi = 0,$$

$$\beta \exp\theta \int_0^1 G(a_1\zeta - c_1)(1 - \zeta)\, d\zeta + e\cos\varphi - q_1 = 0,$$

$$\beta \exp\theta \int_0^1 G(a_2\zeta - c_2)(1 - \zeta)\, d\zeta + e\sin\varphi = 0.$$

In Eqs. (5)-(7)

$$G = [(a_1\zeta - c_1)^2 + (a_2\zeta - c_2)^2]^{\frac{1-n}{2n}}.$$

Passing on to an examination of the heat transfer equation, it can be represented as follows for a screw with i entries:

$$iS(C_0 + \Omega T)\rho \frac{dT}{dx} \int_0^{h_x} W_x\, dz = iS \int_0^{h_x} \left(\tau_{xz} \frac{\partial W_x}{\partial z} + \tau_{yz} \frac{\partial W_y}{\partial z}\right) dz - iS\alpha(T - T_0). \tag{8}$$

By integrating the right and left sides of Eq. (8), taking into account the dimensionless variables (4), we find

$$(1 + \lambda\theta) \frac{d\theta}{d\xi} = -\varkappa\, [\varepsilon(c_1,\ c_2,\ a_1,\ a_2) - a_1 q_1]\ \xi - \mu\theta - \nu. \tag{9}$$

Here $\varkappa = \dfrac{iSA_0V_uH^2k}{C_{in}\rho Q_1 n \tan\delta}$; $\mu = \dfrac{iS\alpha H}{C_{in}\rho Q_1 \tan\delta}$; $\lambda = \dfrac{\Omega n}{kC_{in}}$; $\nu = \dfrac{iS\alpha Hk}{C_{in}\rho Q_1 n \tan\delta}(T_i - T_0)$ are dimensionless complexes; $C_{in} = C_0 + \Omega T_i$ is the heat capacity of the product at the channel input; $\varepsilon(c_1, c_2, a_1, a_2) = (a_1 - c_1)\cos\varphi + (a_2 - c_2)\sin\varphi - e(c_1\cos\varphi + c_2\sin\varphi)$.

In the absence of heat transfer with the ambient $\alpha = \mu = \nu = 0$, but if the heat capacity of the product is virtually independent of temperature, then $\Omega = \lambda = 0$, $C_0 = C_{in} = C$. Under these conditions Eq. (9) is simplified considerably.

Thus, the system of equations Eqs. (7) and (9) must be solved in order to determine c_1, c_2, a_1, a_2, and θ in a random channel cross section.

The pressure gradient and worm-conveyor pump input power are defined as

$$\Delta p = p_{out} - p_{in} = A_0 \int_0^l a_1 dx = \frac{A_0 H}{\tan\delta} \int_1^b a_1 d\xi, \tag{10}$$

$$N = iS\int_0^l (\tau_{xz}|_{z=h_x} V_u \cos\varphi - \tau_{xz}|_{z=0} V_l \cos\varphi + \tau_{yz}|_{z=h_x} V_u \sin\varphi - \tau_{yz}|_{z=0} V_l \sin\varphi)\,dx = -\frac{iSA_0V_uH^2}{\tan\delta}\int_1^b \varepsilon(c_1, c_2, a_1, a_2)\,\xi d\xi. \quad (11)$$

In order to determine the longitudinal pressure gradient at the channel inlet A_0 it must be assumed that $\theta = 0$ and $a_1 = 1$ in the system of equations (7) and then the system of equations must be solved for a given Q_1 in terms of β, c_1, c_2 and a_2. In order to determine the $\xi - \xi_0$ cross section in which $A_1 = 0$ the system equations (7) and (9) must be solved when $a_1 = 0$ in terms of c_1, c_2, a_2, θ, and β. By determining β and ξ_0 we find

$$\xi_0 = \frac{1}{H}\left[\frac{B_1 V_u^n \beta^n}{A_0 \exp(kT_i)}\right]^{\frac{1}{1+n}}.$$

If $\xi_0 \leq b$ ($h_0 \leq h$), then in the whole $1 \geq \xi \geq b$ cross section, $A_1 \geq 0$ ($a_1 \geq 0$). If $\xi_0 \geq b$, then in the $1 \geq \xi \geq \xi_0$ cross section, $A_1 \geq 0$ ($a_1 \geq 0$), and in the $\xi_0 \geq \xi \geq b$ cross section, $A_1 \leq 0$ ($a_1 \leq 0$).

When $e = 0$ ($V_l = 0$) an equation is obtained for a normal worm-conveyor pump with a slowly converging channel.

The following parameter values are selected for specific calculations: $H = 0.005$ m, $h = 0.002$ m, $\varphi = 12°$, $S = 0.097$ m, $i = 1$, $L = 0.3$ m ($l = L/\sin\varphi = 1.42$ m), $e = 0.9$, $V_u = 1$ m/sec, $n = 0.5$, $B_1 = 270$ N·sec^n/m^2, $B = 200$ N·sec^n/m^2, $T_i = 20°$C, $k = 0.015$ 1/deg, $\rho = 970$ kg/m^3, $C_{in} = 1800$ J·kg/deg, $\Omega = 8.4$ J·kg/deg^2, and $p_{in} = 0$.

The problem is solved numerically on a computer. The calculation results are given in Figs. 2-4.

In Fig. 2 the solid lines represent the distribution of pressure along the length of the channel for different values of the product flow rate Q_1 in the case of an adiabatic flow and a product heat capacity independent of temperature ($\alpha = \Omega = \lambda = 0$, $C_0 = C_{in} = C$). It is clear from the figure that the nature of the curves can be different depending on the product flow rate. If the section with a zero longitudinal pressure gradient is located beyond the limits of the channel $\xi_0 \leq 0.4$ (curves 1 and 2), then the pressure rises continuously from the channel input ($\xi = 1$) to its output ($\xi = 0.4$). If $\xi_0 \geq 0.4$ (curves 3 and 4), then the pressure rises at first to a certain magnitude and then falls. The peak on the curves corresponds to the $\xi = \xi_0$ cross section in which $A_1 = 0$.

In the same figure, for purposes of comparison, the dotted lines represent the distribution of pressure along the channel in an isothermal mode of flow. It is clear from the figure that the nature of the curves for isothermal and adiabatic modes of flow (for given flow rates) is maintained and the difference in pressure increases as the product flow rate is reduced. It should be noted that, with the tolerances made in solving the problem for a given product flow rate Q_1 and channel geometry, the positioning of the cross section with zero longitudinal pressure gradient is dependent only on the anomaly in viscosity n and the lead angle of the helical line φ and is independent of whether the process is isothermal or nonisothermal.

In Fig. 3 the solid lines represent the distribution of dimensionless product temperatures Θ along the length of the channel for different flow rate values when $\alpha = \Omega = \lambda = 0$ and when the heat capacity of the product is independent of temperature. As is to be expected, the product heating increases as Q_1 is reduced and the lower the flow rate, the faster the increase. Curves 2a, b, and c are plotted for different values of the coefficient of heat transfer α and different thermal boundary conditions.

It is clear from the figure that when $T_0 > T_i$ (curves 2a and b) the product temperature rises in comparison with the adiabatic flow mode and the greater α, the faster the rise. When $T < T_i$ (curve 2c) the product heating is reduced in comparison with the adiabatic flow mode. In the first case heat flows into the product from the environment and in the second case, in the reverse direction.

The dotted curve 1 is plotted for the case in which the heat capacity of the product increases with a growth in temperature, when, clearly, the product heating is higher. It follows from this that variations in the heat capacity of the product from the temperature in the engineering calculations must not be neglected.

Figure 4 gives the dependences of the dimensionless temperature gradient, and input power on the product flow rate in the adiabatic (solid line) and isothermal (dotted line) modes of flow. It is clear from the figure that in the adiabatic flow mode the product heating increases with a reduction in flow rate tending toward ∞ when $Q_1 = 0$. The pressure gradient and worm-conveyor input power at first grow and then fall with a reduction in the flow rate. This is because, with a low Q_1, the product in the screw channel is heated strongly and its viscosity falls leading to a reduction in the local pressure gradients and pump input power. In an isothermal

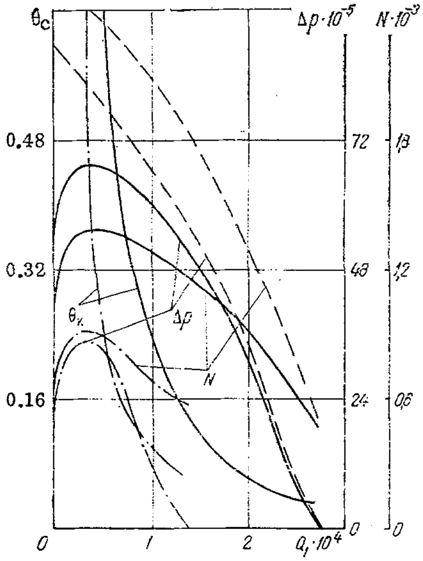

Fig. 4. Dependence of dimensionless temperature θ_C, pressure gradient Δp (N/m^2), and input power N ($N \cdot m/sec$) on product flow rate Q_1 (m^3/sec).

mode of flow the pressure and power grow continuously as the flow rate is reduced, reaching a limit value at $Q_1 = 0$. In the same figure the dashed-dot lines represent the $\theta_C - Q_1$, $\Delta p - Q_1$, and $N - Q_1$ characteristics for a normal worm-conveyor pump ($l = 0$). The nature of the curves is the same as for a belt worm-conveyor pump but the flow rate and pressure gradient for this kind of worm-conveyor are much lower than for the belt-type. It follows from the above that the belt worm-conveyor pump is preferable.

NOTATION

x, y, z, Cartesian coordinates; H, h_x, h, initial, current, and final depth of screw channel; L, screw length; τ_{xz}, τ_{yz}, stress tensor components; l, S, length and width of screw channel, φ, lead angle of helical line; A_1, A_2, pressure gradients; c_1, c_2, integration constants; T_i, T, T_f, initial, current, and final product temperatures; T_0, ambient temperature; θ, dimensionless temperature; p_{in}, p_{out}, product pressures at channel input and output; v_1, v_2, dimensionless velocities; Q_1, Q_2, true product flow rates along and across channel; q_1, q_2, dimensionless product flow rates; i, number of entries in screw; C, heat capacity of product; ρ, density; α, coefficient of heat transfer; N, power; ζ, ξ, dimensionless coordinates; χ, μ, ν, λ, β, dimensionless complexes; $\bar{\tau}$, stress tensor deviator; W_x, W_y, fluid particle velocity projections on x and y axes; $\bar{\Delta}$, strain velocity tensor.

LITERATURE CITED

1. V. I. Yankov, V. I. Boyarchenko, A. L. Krylov, and V. A. Bratukhina, Inzh.-Fiz. Zh., 28, No. 3, (1975).
2. L. M. Beder, Khim. Volokna, No. 6 (1969).
3. L. M. Beder, in: Synthetic Polymer Fibers [in Russian], Khimiya, Moscow (1970).
4. S. A. Bostandzhiyan, V. I. Boyarchenko, and G. N. Kargapolova, Inzh.-Fiz. Zh., 21, No. 2 (1971).